CIS 5560

Cryptography
Lecture 19

Course website:
pratyushmishra.com/classes/cis-5560-s25

Slides adapted from Dan Boneh and Vinod Vaikuntanathan

Recap of last lecture

New primitive: Digital Signatures

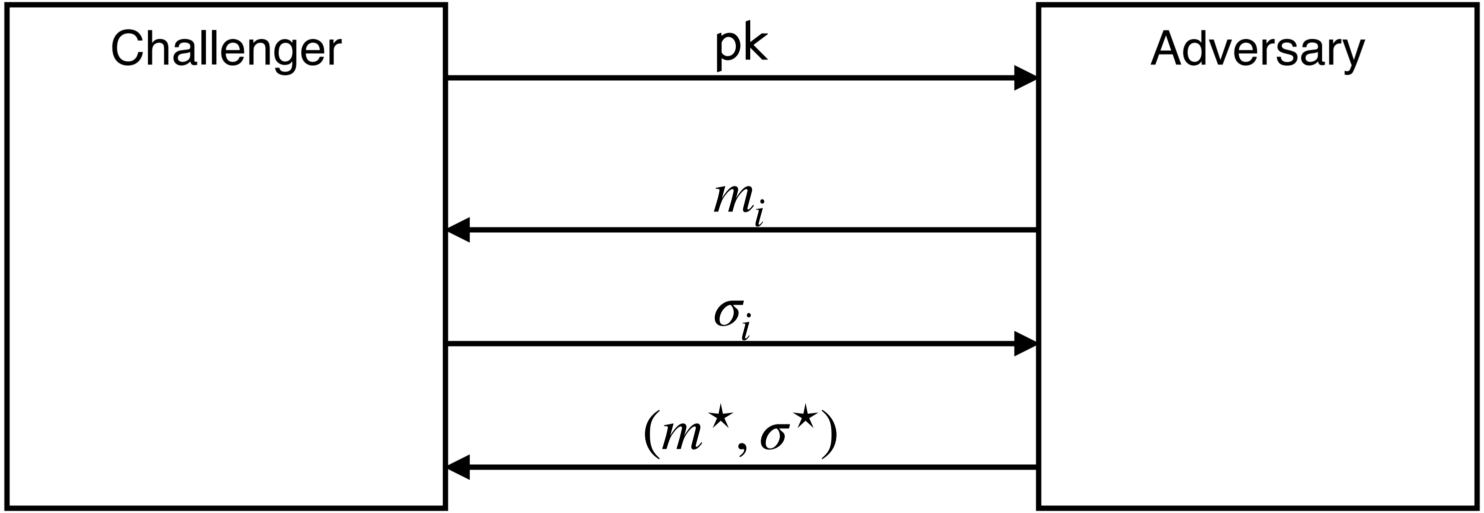
Digital Signatures: Definition

A triple of PPT algorithms $(\mathsf{Gen}, \mathsf{Sign}, \mathsf{Verify})$ such that

- Key generation: $\mathsf{Gen}(1^n) \to (\mathsf{sk}, \mathsf{pk})$
- Message signing: $\mathsf{Sign}(\mathsf{sk}, m) \to \sigma$
- Signature verification: $\mathsf{Verify}(\mathsf{pk}, m, \sigma) \to b \in \{0,1\}$

Correctness: For all vk, sk, m: $\mathsf{Verify}(\mathsf{pk}, m, \mathsf{Sign}(\mathsf{sk}, m)) = 1$

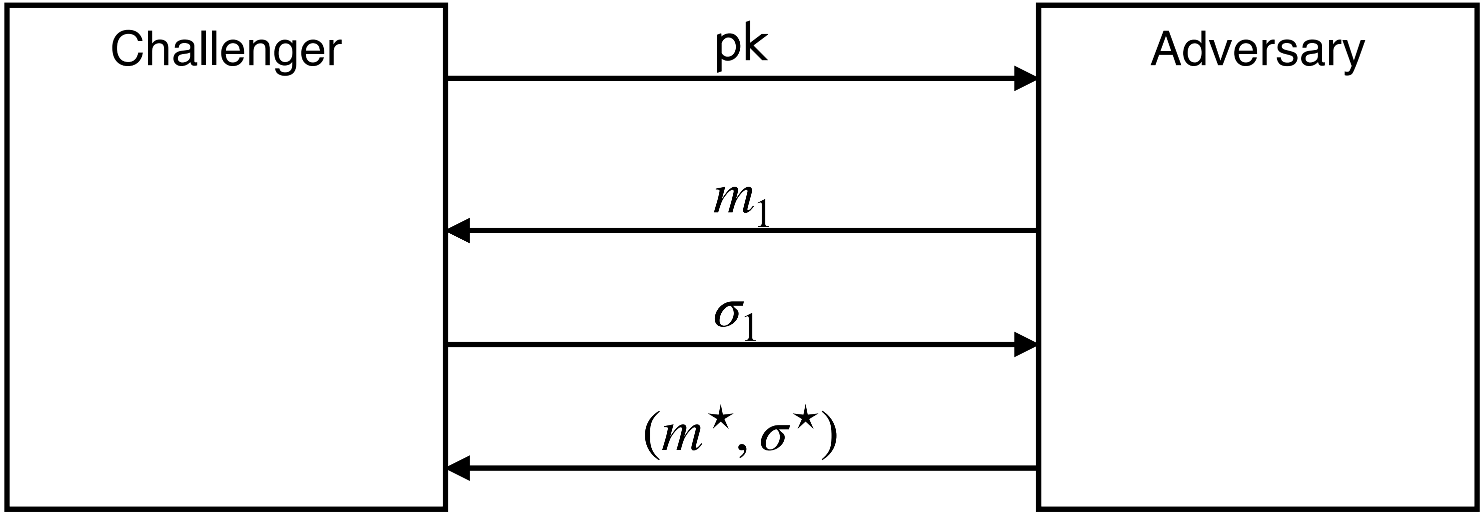
EUF-CMA for Signatures



$$\Pr\left[\begin{array}{c} m^\star \notin \{m_i\} \\ \text{and} \\ \mathsf{Verify}(\mathsf{pk}, m^\star, \sigma^\star) = 1 \end{array}\right] = \mathsf{negl}(\lambda)$$

Today's lecture

Simpler Goal: EUF-CMA for *1-time Signatures*



$$\Pr\left[\begin{array}{c} m^\star \neq m_1 \\ \text{and} \\ \mathsf{Verify}(\mathsf{pk}, m^\star, \sigma^\star) = 1 \end{array}\right] = \mathsf{negl}(\lambda)$$

Lamport (One-time) Signatures from OWFs

Signing Key sk: $\begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$

Public Key pk: $\begin{pmatrix} y_0 = f(x_0) \\ y_1 = f(x_1) \end{pmatrix}$

Signing a bit b: The signature is $\sigma = x_b$

Verifying (b, σ): Check if $f(\sigma) = y_b$

Claim: Assuming f is a OWF, no PPT adversary can produce a signature of $\bar{b}$ given a signature of b.

Lamport One-time Signatures for n-bit msgs

Secret Key sk: $\begin{pmatrix} x_{1,0} & x_{2,0} & \cdots & x_{n,0} \\ x_{1,1} & x_{1,1} & \cdots & x_{n,1} \end{pmatrix}$

Public Key pk: $\begin{pmatrix} y_{1,0} & y_{2,0} & \cdots & y_{n,0} \\ y_{1,1} & y_{2,1} & \cdots & y_{n,1} \end{pmatrix}$ where $y_{i,b} = f(x_{i,b})$.

Signing $m = (m_1, \ldots, m_n)$: $\sigma = (x_{1,m_1}, x_{2,m_2}, \ldots, x_{n,m_n})$

Claim: Assuming f is a OWF, no PPT adv can produce a signature of m given a signature of a single $m' \neq m$.

Claim: Can forge signature on any message given the signatures on (some) two messages.

Lamport (One-time) Signatures for arbitrary bits

Secret Key sk: $\begin{pmatrix} x_{1,0} & x_{2,0} & \cdots & x_{n,0} \\ x_{1,1} & x_{1,1} & \cdots & x_{n,1} \end{pmatrix}$

Public Key pk: $\begin{pmatrix} y_{1,0} & y_{2,0} & \cdots & y_{n,0} \\ y_{1,1} & y_{2,1} & \cdots & y_{n,1} \end{pmatrix}$ where $y_{i,b} = f(x_{i,b})$.

Signing m:

1. $z := H(m)$
2. $\sigma = (x_{1,z_1}, x_{2,z_2}, \ldots, x_{n,z_n})$

Claim: Assuming H is CRH and f is a OWF, no PPT adv can produce a signature of m given a signature of a single $m' \neq m$.

Claim: Can forge signature on any message given the signatures on (some) two messages.

So far, only one-time security…

Constructing a Signature Scheme

Step 0. Still one-time, but arbitrarily long messages.

Step 1. Many-time: Stateful, Growing Signatures.

Step 2. How to Shrink the signatures.

Step 3. How to Shrink Alice's storage.

Step 4. How to make Alice stateless.

Step 5 (*optional*). How to make Alice stateless and deterministic.

Constructing a Signature Scheme

Theorem [Naor-Yung’89, Rompel’90]
(EUF-CMA-secure) Signature schemes exist assuming that one-way functions exist.

TODAY:
(EUF-CMA-secure) Signature schemes exist assuming that collision-resistant hash functions exist.

(Many-time) Signature Scheme

In four+ steps

Step 1. Stateful, Growing Signatures. Idea: Signature ***Chains***

Step 2. How to Shrink the signatures. Idea: Signature ***Trees***

Step 3. How to Shrink Alice's storage.
Idea: ***Pseudorandom Trees***

Step 4. How to make Alice stateless.
Idea: ***Randomization***

Step 5 (*optional*). How to make Alice stateless and deterministic. Idea: ***PRFs.***

Step 1: Stateful Many-time Signatures

Idea: Signature Chains.

Alice starts with a secret signing Key sk_0

When signing a message m_1:

Generate a new pair $(\mathsf{sk}_1, \mathsf{pk}_1)$

Produce signature $\sigma_1' \leftarrow \mathsf{Sign}(\mathsf{sk}_0, m_1 || \mathsf{pk}_1)$

Output $\mathsf{pk}_1 || \sigma_1'$.

Remember $\mathsf{pk}_1 || m_1 || \sigma_1$ as well as sk_1.

To verify a signature $\mathsf{pk}_1 || \sigma_1$ for message m_1:

Run $\mathsf{Verify}(\mathsf{pk}_0, \mathsf{pk}_1 || m_1, \sigma_1') = 1$

Step 1: Stateful Many-time Signatures

Idea: Signature Chains.

Alice starts with a secret signing Key sk_0

When signing a message m_1:

- Generate a new pair $(\mathsf{sk}_1, \mathsf{pk}_1)$
- Produce signature $\sigma_1 \leftarrow \mathsf{Sign}(\mathsf{sk}_0, m_1 || \mathsf{pk}_1)$
- Output $\mathsf{pk}_1 || \sigma_1$.
- Remember $\mathsf{pk}_1 || m_1 || \sigma_1$ as well as sk_1.

$$\mathsf{pk}_0 \xrightarrow{\sigma_1} \overset{m_1}{\mathsf{pk}_1}$$

Step 1: Stateful Many-time Signatures

Idea: Signature Chains.

Alice starts with a secret signing Key sk_0

When signing the next message m_2

Generate a new pair $(\mathsf{sk}_2, \mathsf{pk}_2)$

Produce signature $\sigma_2 \leftarrow \mathsf{Sign}(\mathsf{sk}_1, m_2 || \mathsf{pk}_2)$

Output ???

$$\begin{array}{ccc} & & m_1 \\ \mathsf{pk}_0 & \xrightarrow{\sigma_1} & \mathsf{pk}_1 \end{array}$$

Step 1: Stateful Many-time Signatures

Idea: Signature Chains.

Alice starts with a secret signing Key sk_0

When signing the next message m_2

Generate a new pair $(\mathsf{sk}_2, \mathsf{pk}_2)$

Produce signature $\sigma_2 \leftarrow \mathsf{Sign}(\mathsf{sk}_1, m_2 || \mathsf{pk}_2)$

Output $\mathsf{pk}_2 || \sigma_2$??

$$\mathsf{pk}_0 \xrightarrow{\sigma_1} \overset{m_1}{\mathsf{pk}_1}$$

Step 1: Stateful Many-time Signatures

Idea: Signature Chains.

Alice starts with a secret signing Key sk_0

When signing the next message m_2

Generate a new pair $(\mathsf{sk}_2, \mathsf{pk}_2)$

Produce signature $\sigma_2 \leftarrow \mathsf{Sign}(\mathsf{sk}_1, m_2 || \mathsf{pk}_2)$

Output $\mathsf{pk}_1 || \mathsf{pk}_2 || \sigma_2$??

$$\mathsf{pk}_0 \xrightarrow{\sigma_1} \overset{m_1}{\mathsf{pk}_1}$$

Step 1: Stateful Many-time Signatures

Idea: Signature Chains.

Alice starts with a secret signing Key sk_0

When signing the next message m_2

Generate a new pair $(\mathsf{sk}_2, \mathsf{pk}_2)$

Produce signature $\sigma_2 \leftarrow \mathsf{Sign}(\mathsf{sk}_1, m_2 || \mathsf{pk}_2)$

Output $(\mathsf{pk}_1 || m_1 || \sigma_1) || \mathsf{pk}_2 || \sigma_2$

(additionally) remember $\mathsf{pk}_2 || m_2 || \sigma_2$ as well as sk_2.

$$\mathsf{pk}_0 \xrightarrow{\sigma_1} \overset{m_1}{\mathsf{pk}_1} \xrightarrow{\sigma_2} \overset{m_2}{\mathsf{pk}_2}$$

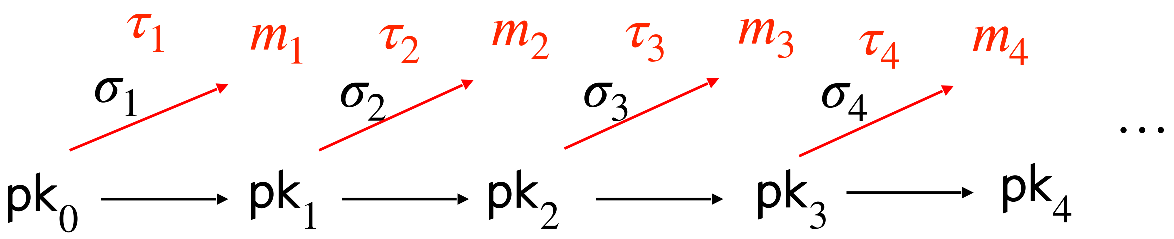
Step 1: Stateful Many-time Signatures

Idea: Signature Chains.

Two major problems:

1. Alice is *stateful*: Alice needs to remember a whole lot of things, $O(T)$ information after T steps.

2. The *signatures grow*: Length of the signature of the T-th message is $O(T)$.



(Many-time) Signature Scheme

In four+ steps

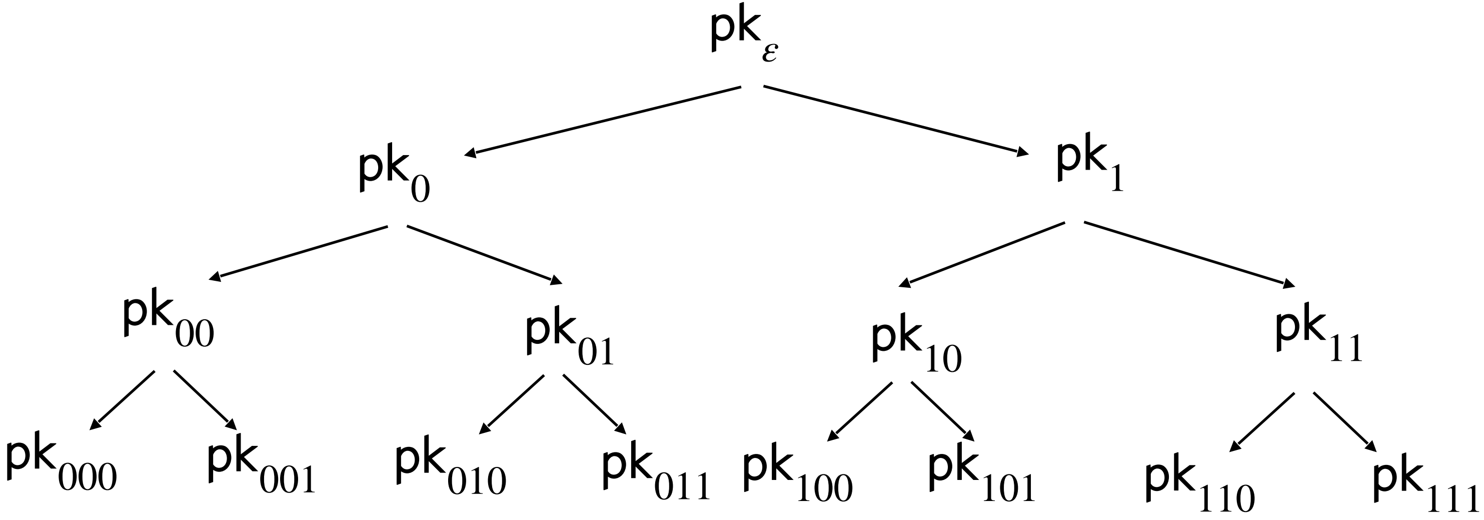
Step 1. Stateful, Growing Signatures. Idea: Signature ***Chains***

Step 2. How to Shrink the signatures. Idea: Signature ***Trees***

Step 2: Shrinking signatures

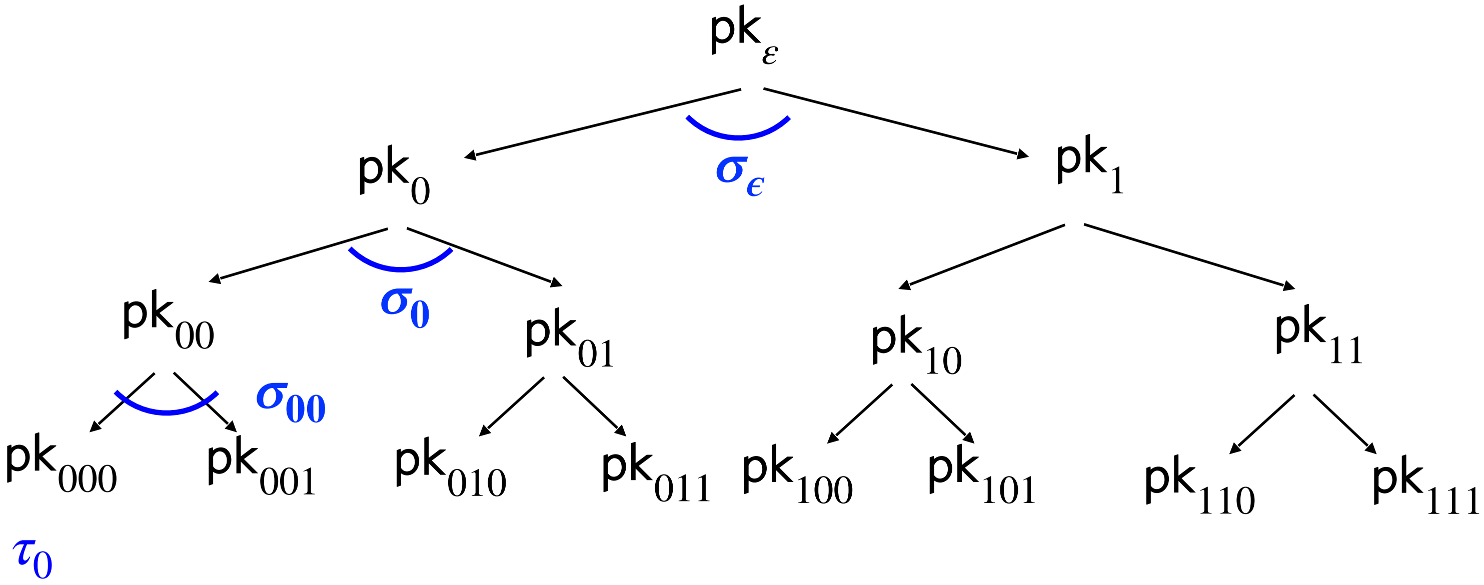
$\mathsf{pk}_{\mathcal{E}}$

Step 2: Shrinking signatures



Alice (the *stateful* signer) computes many $(\mathsf{pk}, \mathsf{sk})$ pairs and arranges them in a tree of depth = sec. param. λ

Step 2: Shrinking signatures

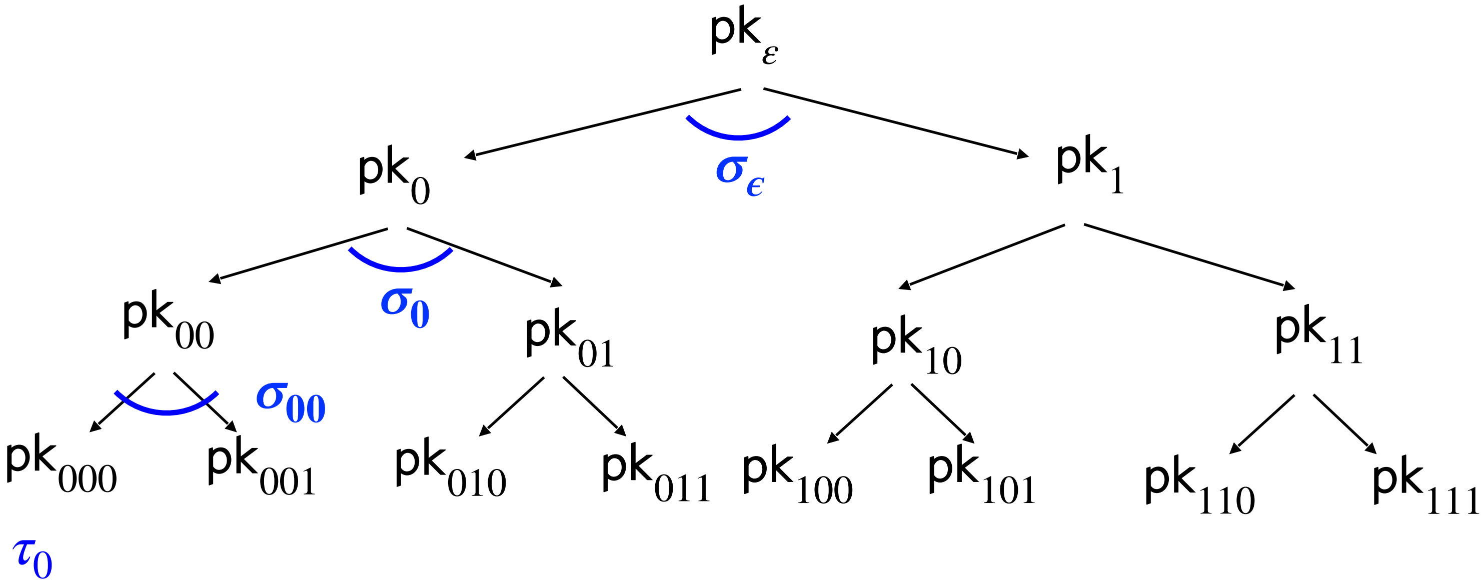


Signature of the zeroth message m_0:

Use sk_{000} to sign m_0.

"Authenticate" pk_{000} using the "signature path".

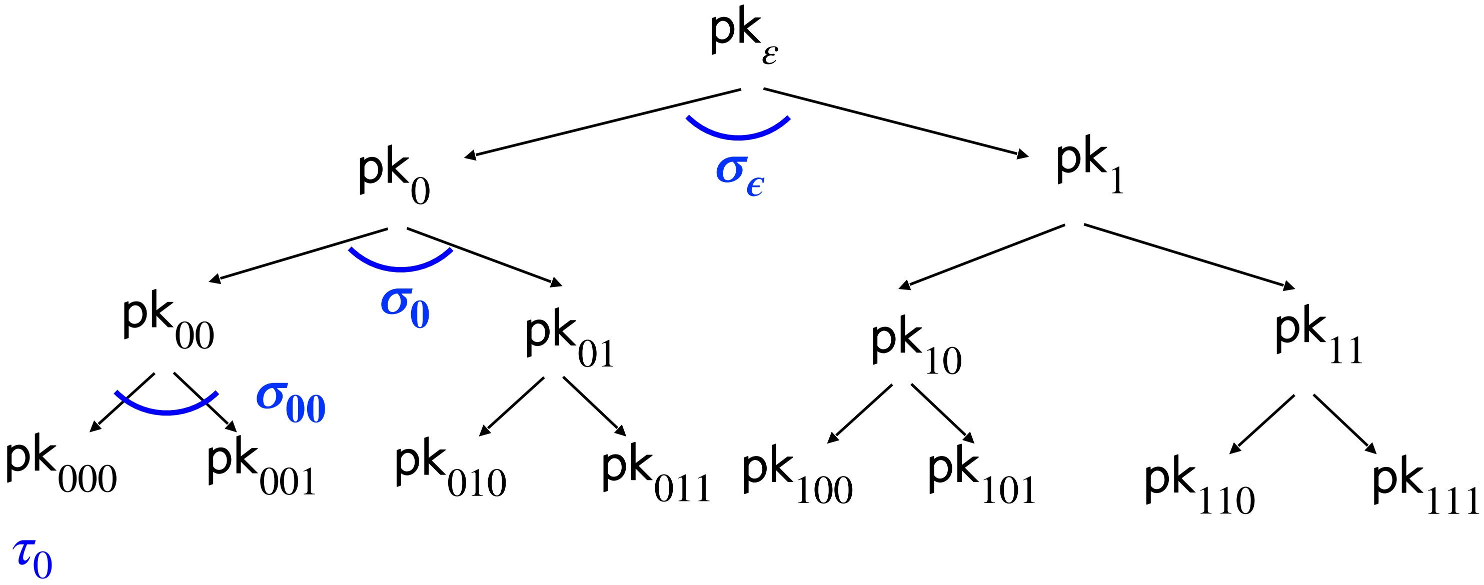
Step 2: Shrinking signatures



Signature of the zeroth message m_0:

$\big(\sigma_\varepsilon \leftarrow \mathsf{Sign}(\mathsf{sk}_\epsilon, \mathsf{pk}_0 \| \mathsf{pk}_1),\ \sigma_0 \leftarrow \mathsf{Sign}(\mathsf{sk}_0, \mathsf{pk}_{00} \| \mathsf{pk}_{01}),$
$\sigma_{00} \leftarrow \mathsf{Sign}(\mathsf{sk}_{00}, \mathsf{pk}_{000} \| \mathsf{pk}_{001}),\ \tau_0 \leftarrow \mathsf{Sign}(\mathsf{sk}_{000}, m_0)\big)$

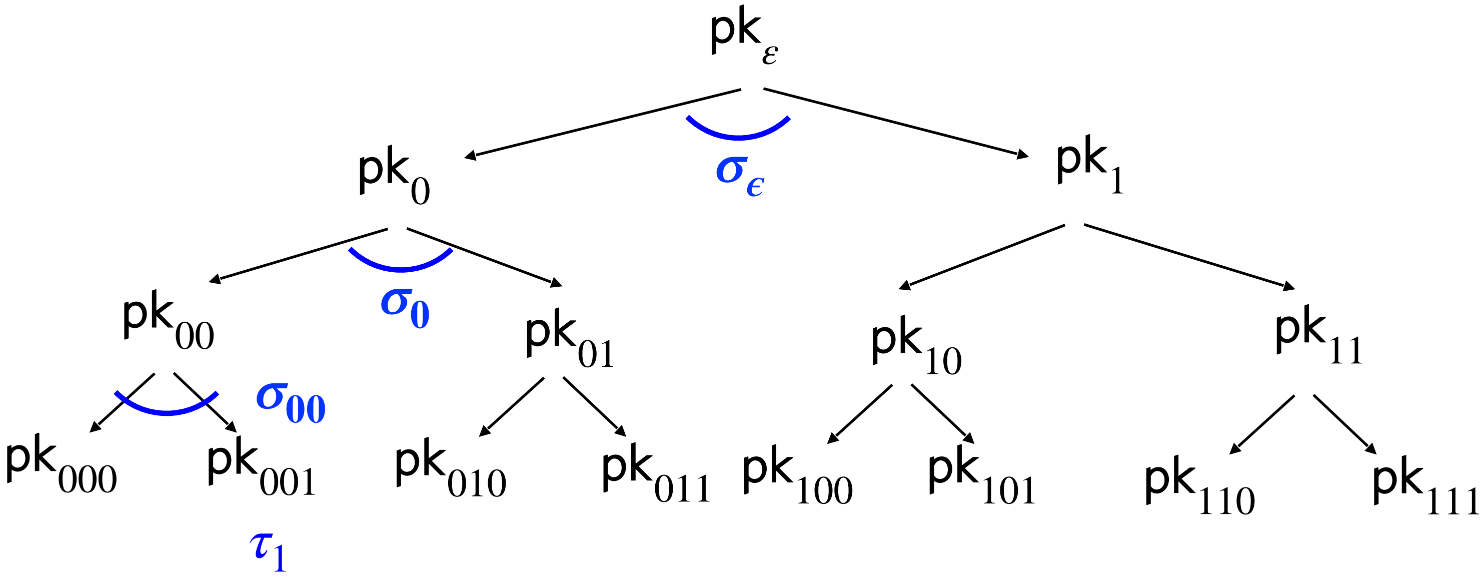
Step 2: Shrinking signatures



Signature of the zeroth message $\boldsymbol{m_0}$:

(Authentication path for pk_{000}, $\tau_0 \leftarrow \mathsf{Sign}(\mathsf{sk}_{000}, m_0)$)

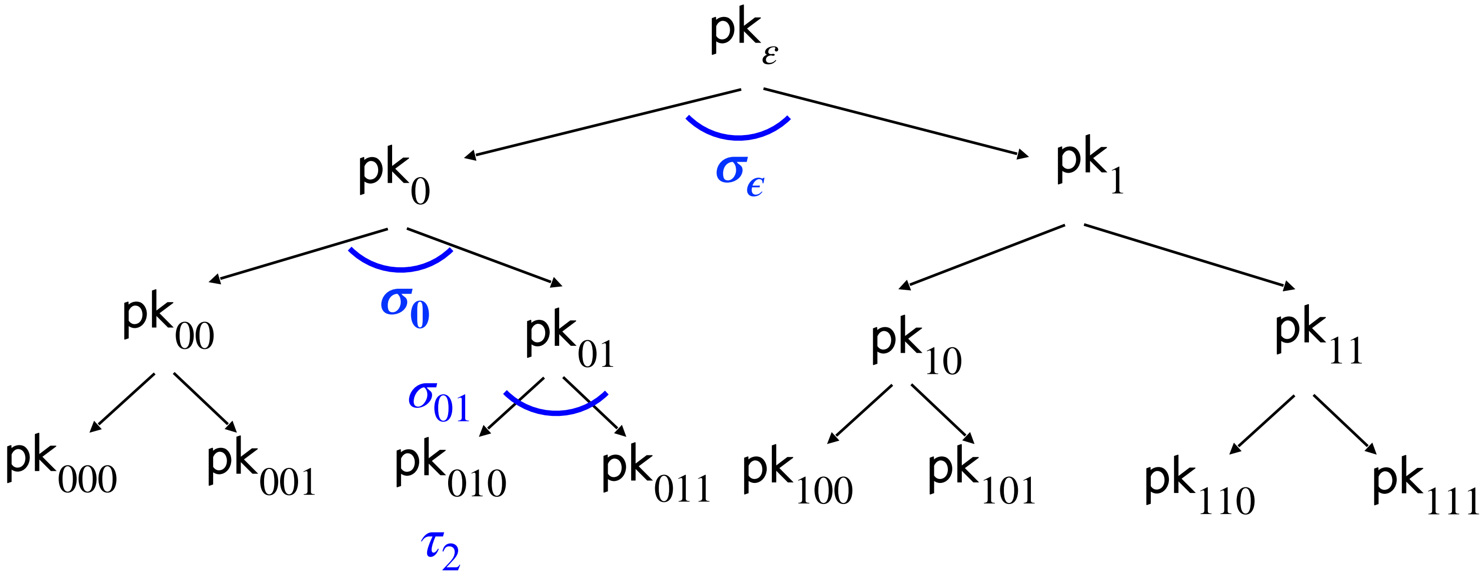
Step 2: Shrinking signatures



Signature of message m_1

$\Big($Authentication path for pk_{001}, $\tau_1 \leftarrow \mathsf{Sign}(\mathsf{sk}_{001}, m_1)\Big)$

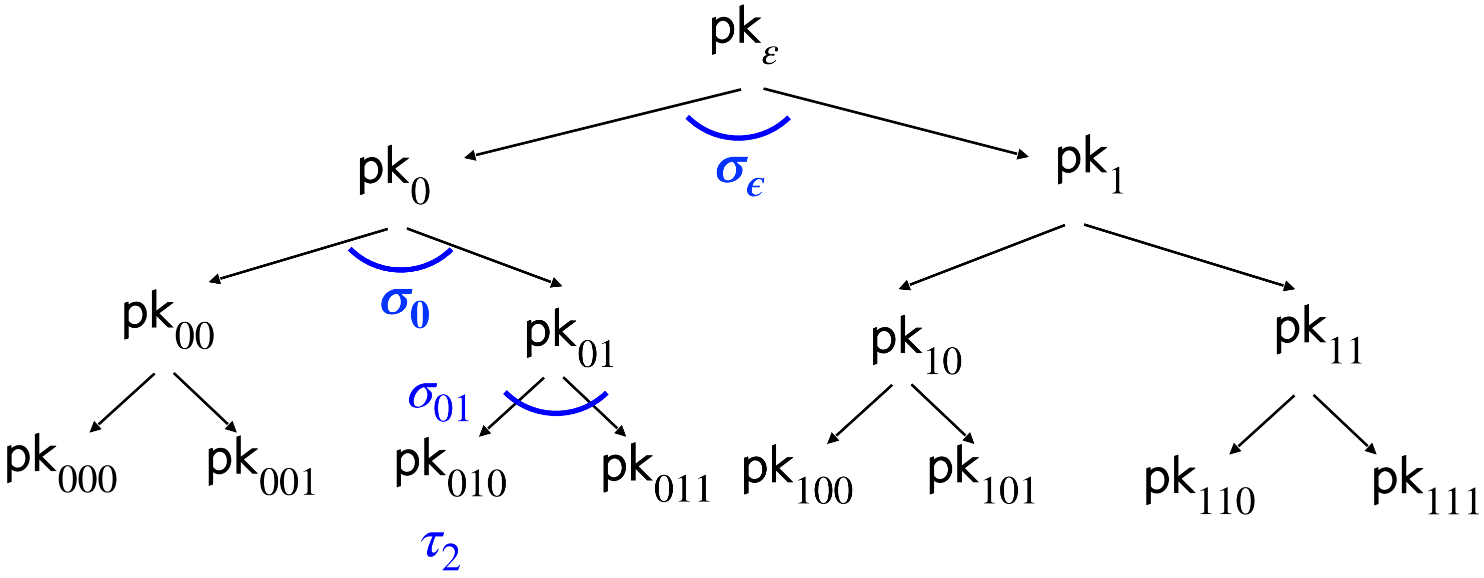
Step 2: Shrinking signatures



Signature of message m_2

(Authentication path for pk_{010}, $\tau_2 \leftarrow \mathsf{Sign}(\mathsf{sk}_{010}, m_2)$)

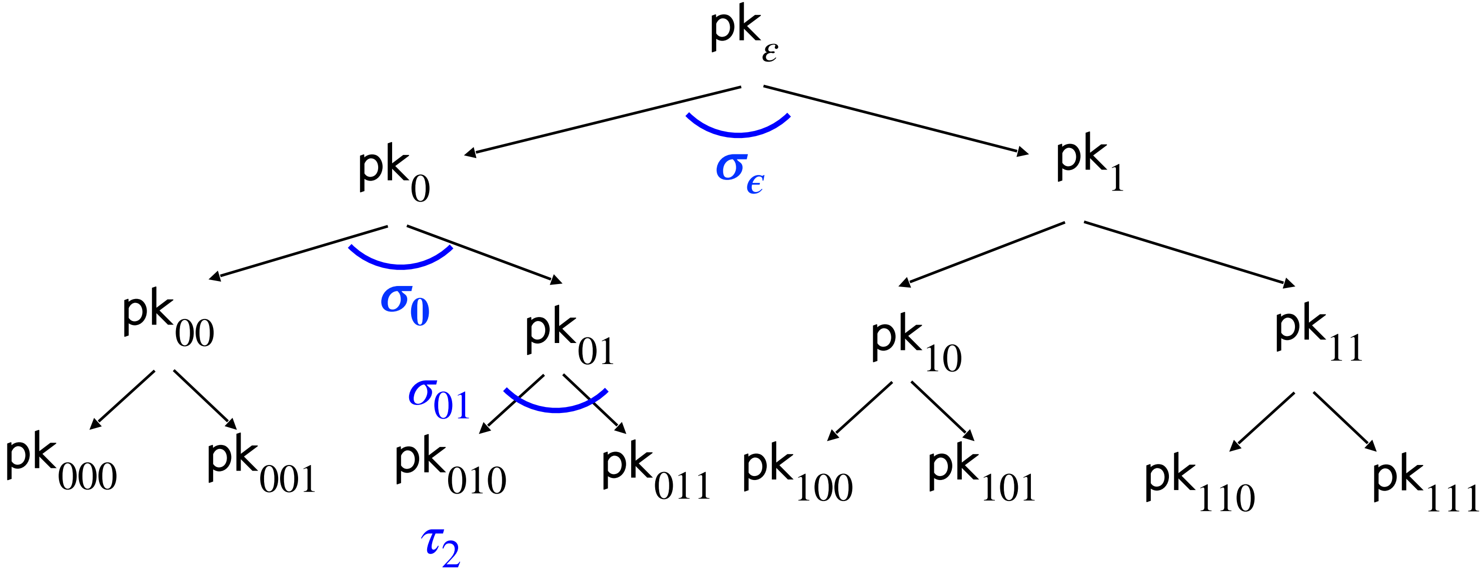
Step 2: Shrinking signatures



GOOD NEWS: 

Each verification key (incl. at the leaves) is used only once, so one-time security suffices!

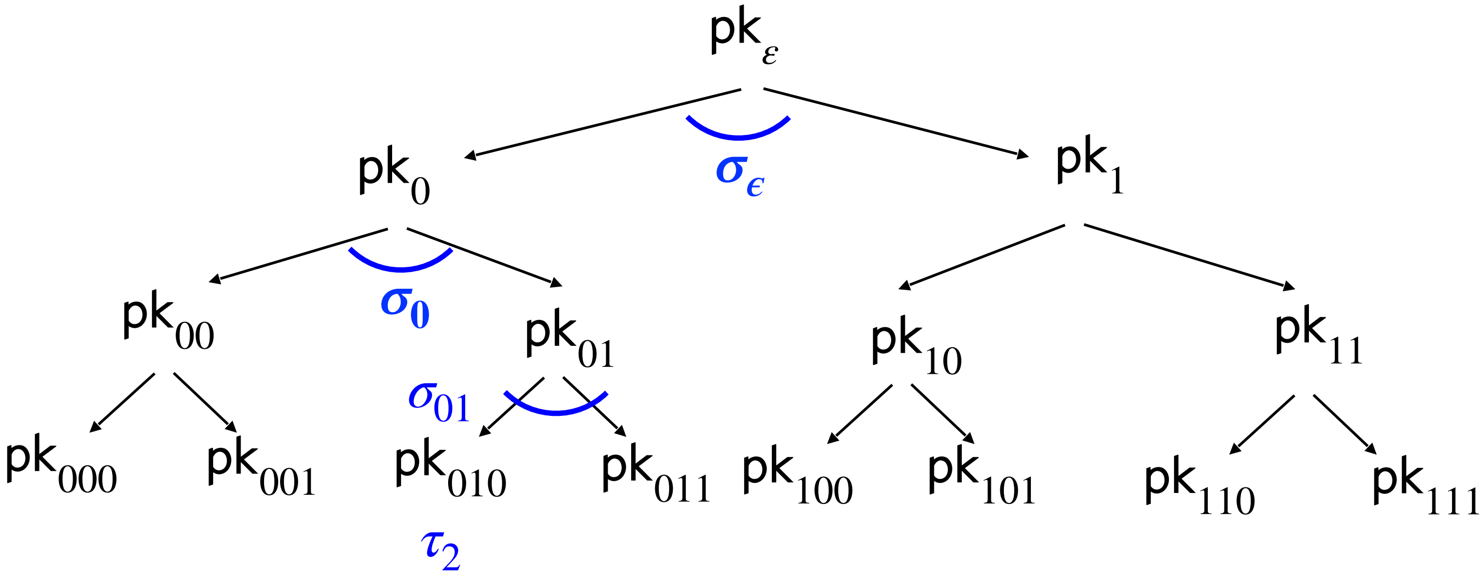
Step 2: Shrinking signatures



GOOD NEWS:

Signatures consist of λ one-time signatures and do now grow with time!

Step 2: Shrinking signatures



BAD NEWS: 

Signer generates and keeps the entire ($\approx\ 2^{\lambda}$-size) signature tree in memory!

(Many-time) Signature Scheme

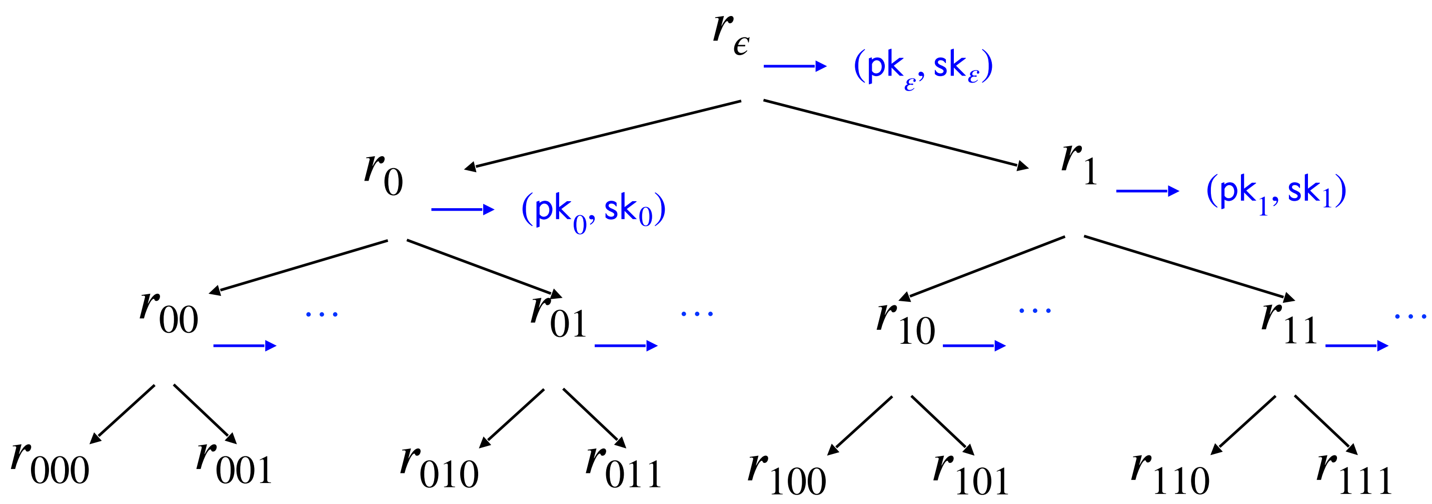
In four+ steps

Step 1. Stateful, Growing Signatures. Idea: Signature ***Chains***

Step 2. How to Shrink the signatures. Idea: Signature ***Trees***

Step 3. How to Shrink Alice's storage.
Idea: ***Pseudorandom Trees***

Step 3: Pseudorandom signature trees



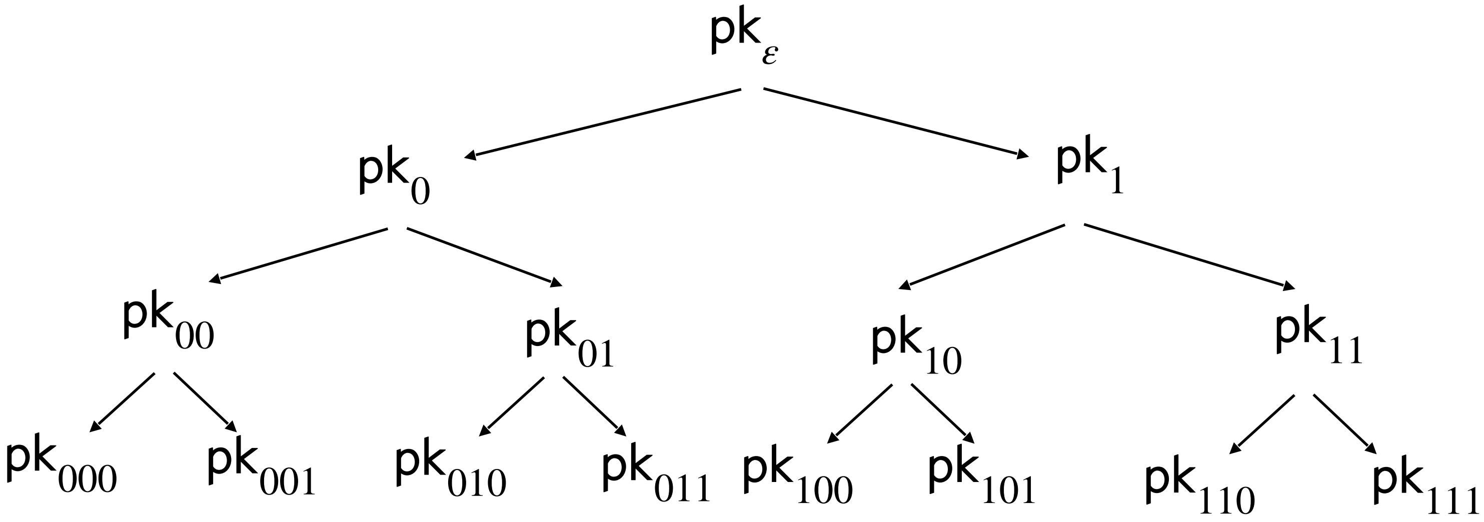
Tree of pseudorandom values:

The signing key is a PRF key k.

"Lazily" populate the nodes with $r_x := \mathsf{PRF}(k, x)$.

Use r_x to derive the keys $(\mathsf{pk}_x, \mathsf{sk}_x) \leftarrow \mathsf{Gen}(1^\lambda; r_x)$.

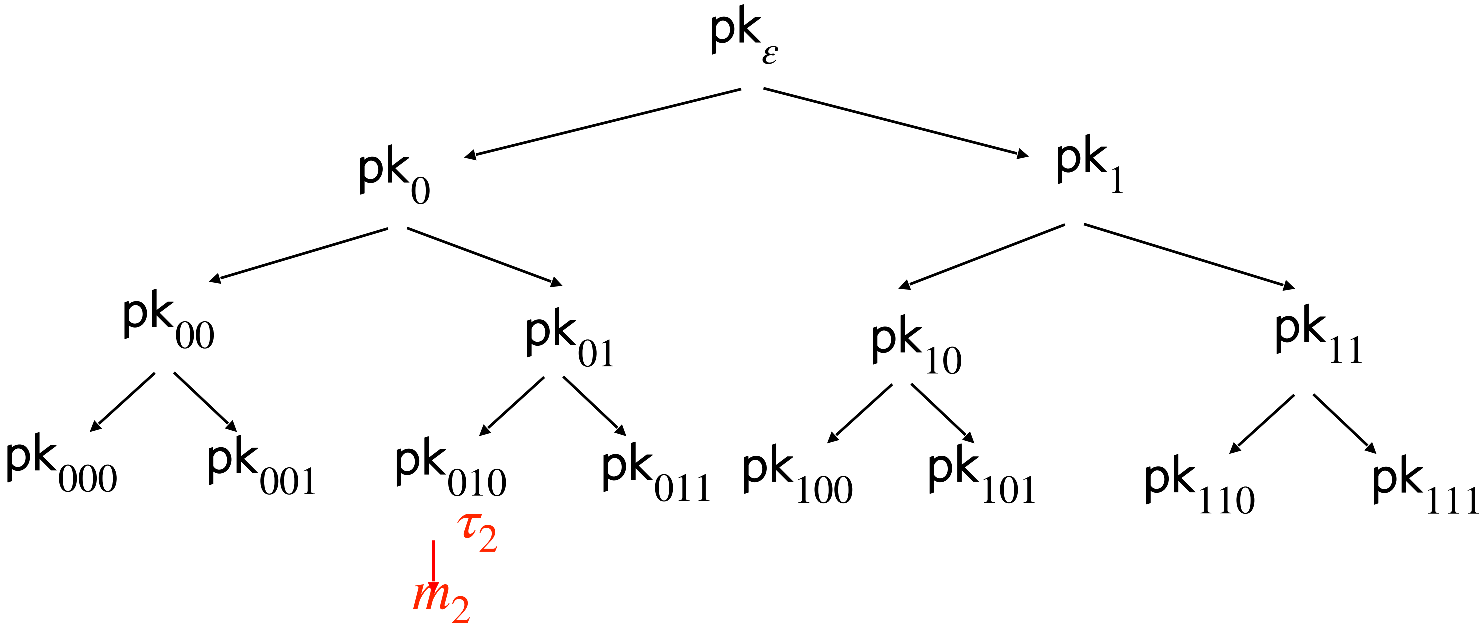
Step 3: Pseudorandom signature trees



GOOD NEWS: 

Short signatures and small storage for the signer

Step 3: Pseudorandom signature trees



BAD NEWS:

Signer needs to keep a counter indicating which ***leaf*** (which tells her which secret key) to use next.

(Many-time) Signature Scheme

In four+ steps

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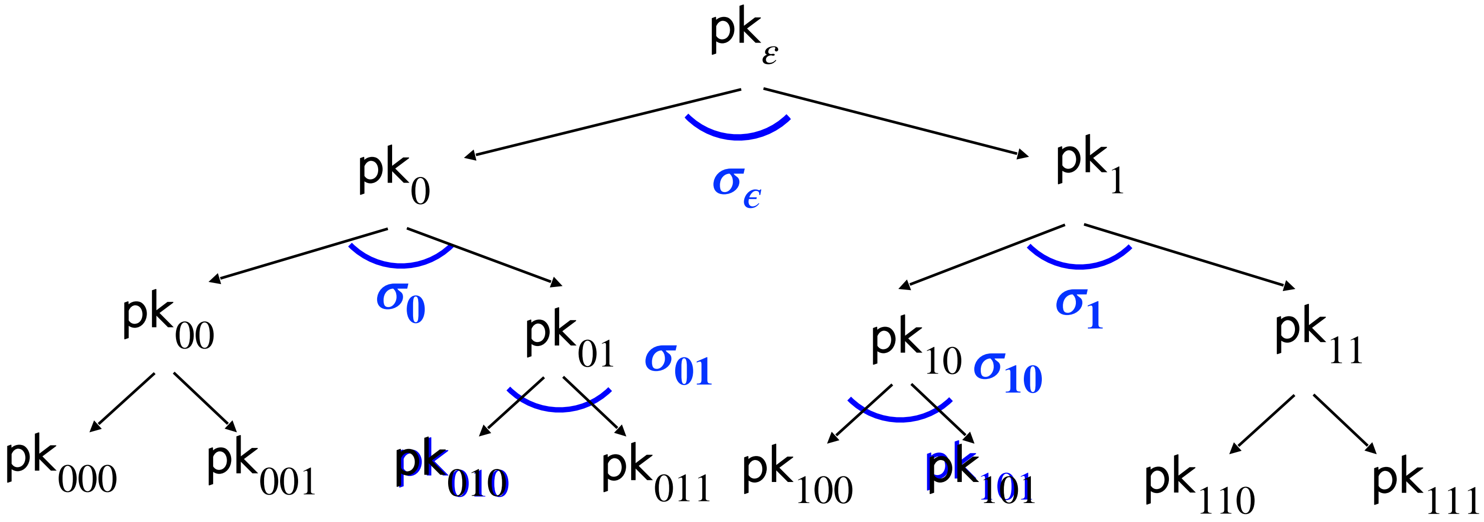
Step 4. How to make Alice stateless.
Idea: ***Randomization***

Step 4: Statelessness via randomization



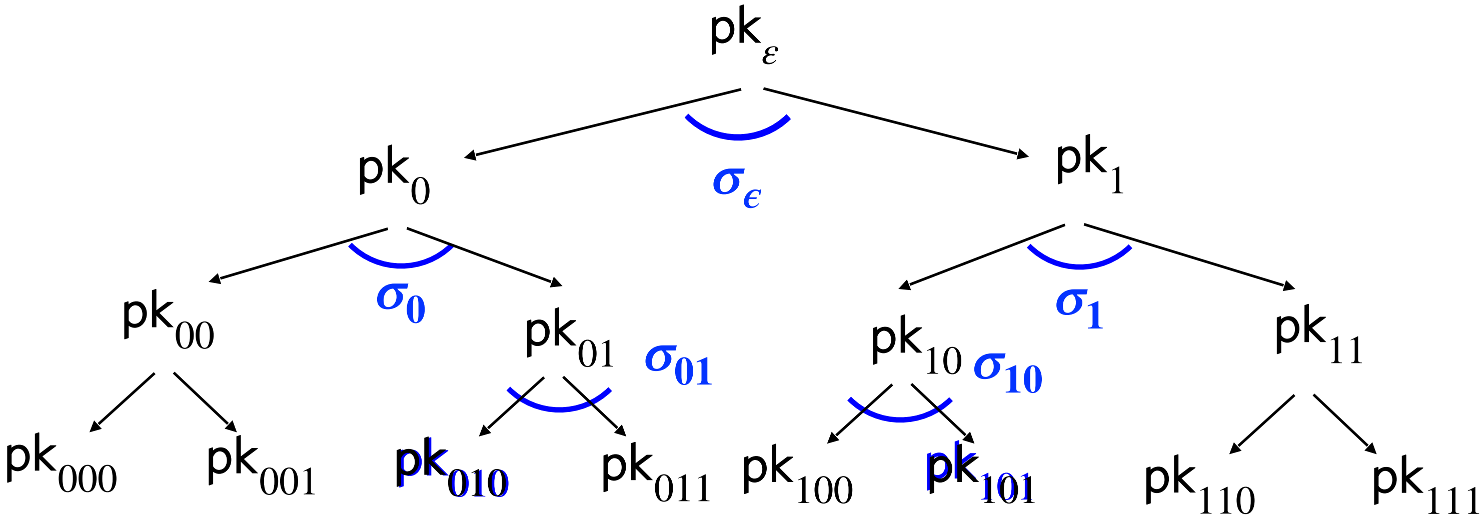
Signature of a message m:

Pick a **random** leaf r. Use pk_r to sign m.

$\sigma_r \leftarrow \mathsf{Sign}(\mathsf{sk}_r, m)$

Output $(r, \sigma_r,$ authentication path for $\mathsf{pk}_r)$

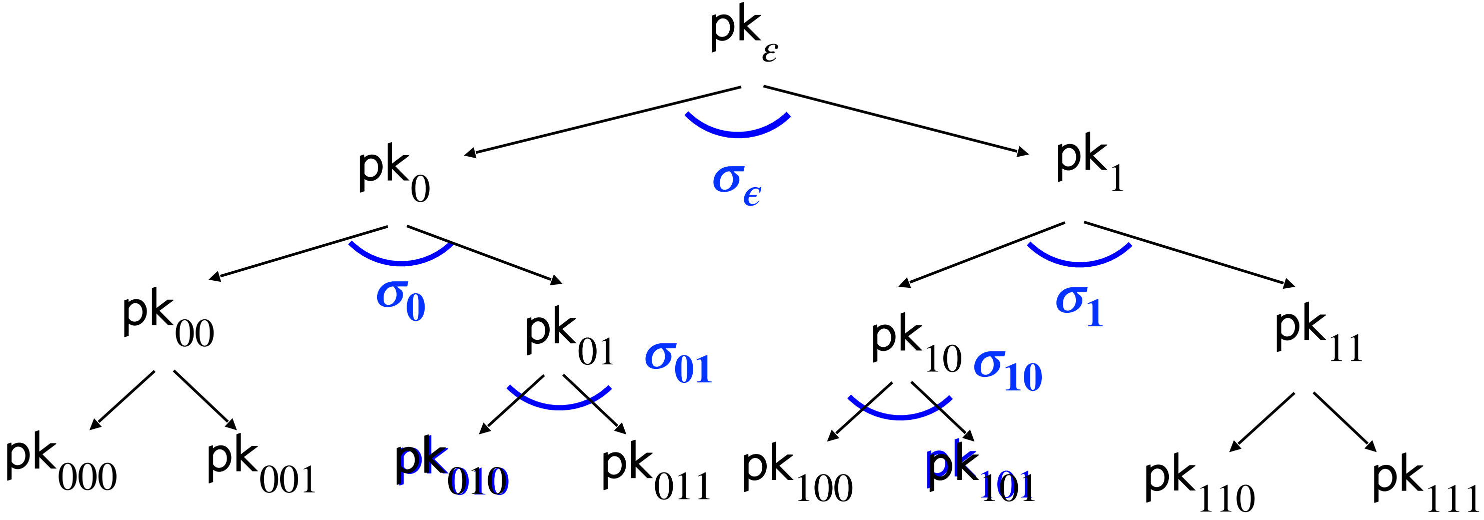
Step 4: Statelessness via randomization



GOOD NEWS: 

No need to keep state.

Step 4: Statelessness via randomization



Key Idea:

If the signer produces q signatures, the probability she picks the same leaf twice is $\leq q^2/2^\lambda$.

(Many-time) Signature Scheme

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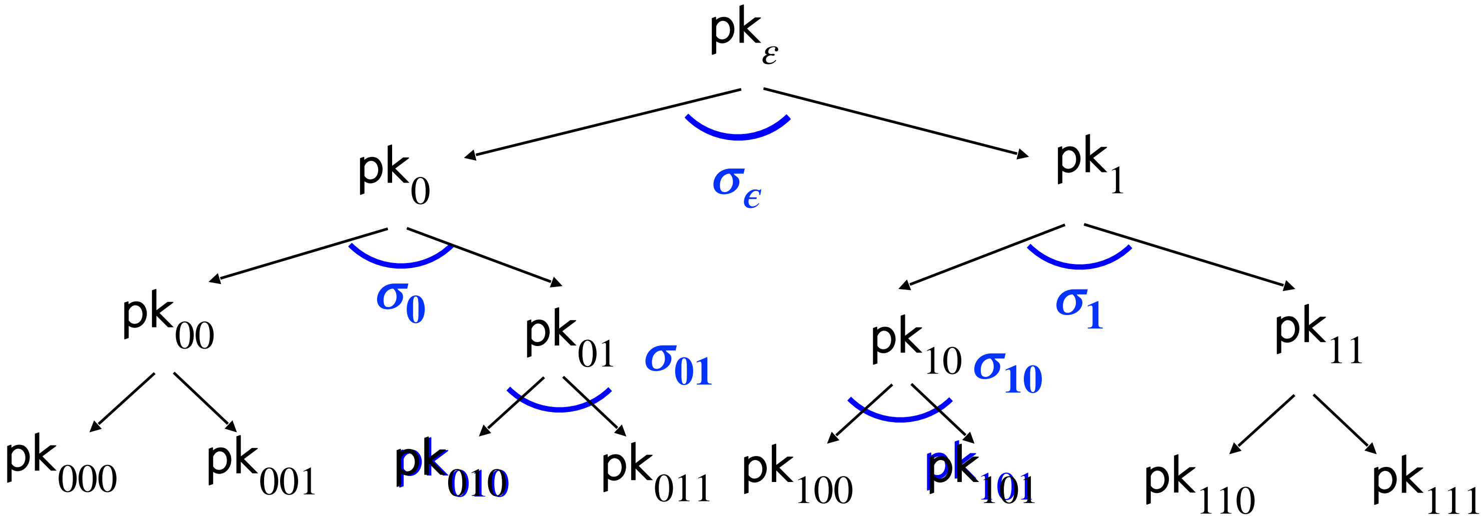
Step 4. How to make Alice stateless.
Idea: ***Randomization***

Step 5 (*optional*). How to make Alice stateless and deterministic. Idea: ***PRFs.***

Step 4: Deterministic signer



Key Idea:

Generate r pseudo-randomly.

Have another PRF key k' and let $r = \mathsf{PRF}(k', m)$

That’s it for the construction.

Digital Signature Construction

- Historically regarded as inefficient; therefore, never used in practice.

- However, this signature scheme (or variants thereof) are now called “hash-based signatures” and seeing a re-emergence as a candidate post-quantum secure signature scheme. E.g. https://sphincs.org/