CIS 5560

Cryptography
Lecture 17

Course website:
pratyushmishra.com/classes/cis-5560-s24/

Slides adapted from Dan Boneh and Vinod Vaikuntanathan

Recap of Last Lecture(s)

- Public Key Encryption
  - Definition of IND-CPA
- ElGamal Encryption
  - Version with message space = $\mathbb{G}$
  - Version with arbitrary message space
- Public Key Encryption from **Trapdoor OWFs**
  - RSA Encryption
    - Arithmetic modulo composites
    - Factoring

Today's Lecture

- Integrity for public key encryption
  - IND-CCA
  - Construction of IND-CCA

Public key encryption

Def: a public-key encryption system is a triple of algs. (G, E, D)

- Gen(): randomized alg. outputs a key pair (pk, sk)
- Enc(pk, m): randomized alg. that takes $m \in \mathcal{M}$ and outputs $c \in \mathcal{C}$
- Dec(sk, c): deterministic alg. that takes $c \in \mathcal{C}$ and outputs $m \in \mathcal{M} \cup \{ \perp \}$

Correctness: $\forall$(pk, sk) output by Gen(), $\forall m \in \mathcal{M}$, Dec(sk, Enc(pk, m)) $= m$

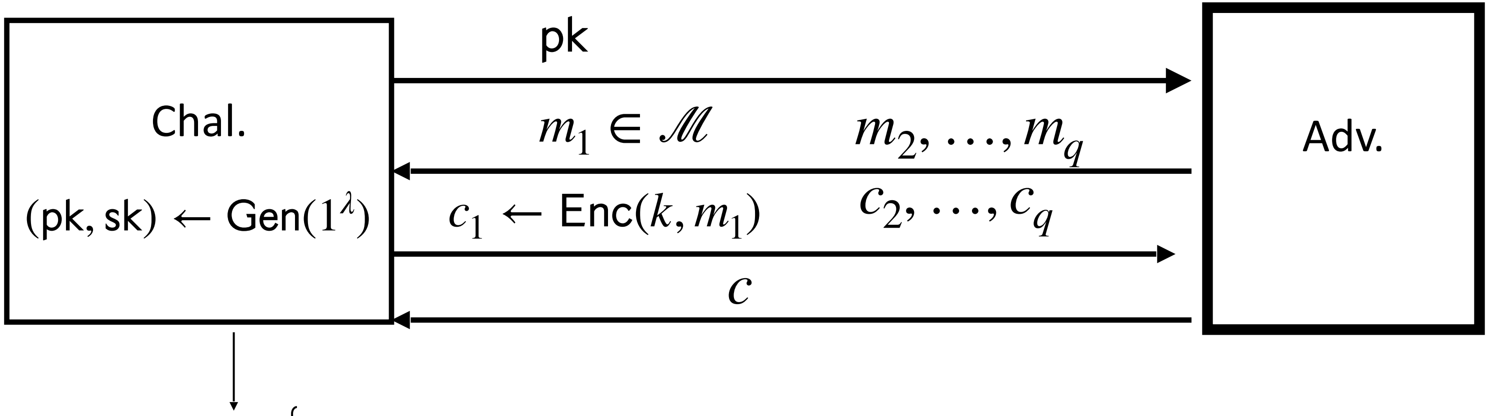
Security: IND-CPA for PKE

For all PPT adversaries $\mathscr{A}$, the following holds:

$$\Pr\left[b = \mathscr{A}(\mathsf{Enc}(\mathsf{pk}, m_b)) \,\middle|\, \begin{array}{r} (\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{Gen}(1^n) \\ \text{Sample } b \leftarrow \{0,1\} \\ (m_0, m_1) \leftarrow \mathscr{A}(\mathsf{pk}) \end{array} \right] \leq \mathsf{negl}(n)$$

What about security against active attacks?

Can we achieve ciphertext integrity?



$$b \begin{cases} b = 1 & \text{if } \mathsf{Dec}(k, c) \neq \perp \quad \text{and } c \notin \{c_1, \ldots, c_q\} \\ b = 0 & \text{otherwise} \end{cases}$$

Def: (Gen, Enc, Dec) has **ciphertext integrity** if for all PPT A:

$$\mathsf{Adv}_{\mathrm{CI}}[A] = \Pr[b = 1] = \mathsf{negl}(\lambda)$$

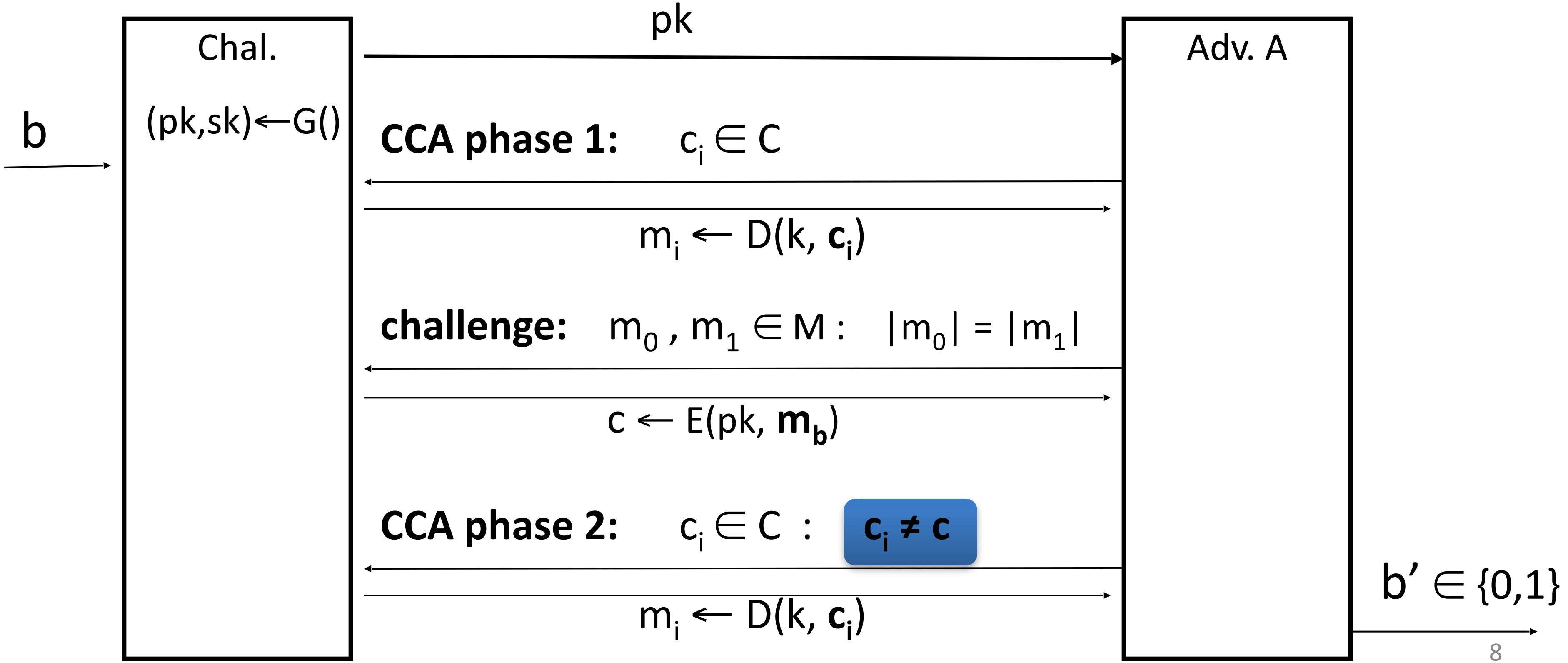
Problem

In public-key settings:

- Attacker **can** *always* create new ciphertexts using pk !!
- So instead: we directly require chosen ciphertext security

PKE Chosen Ciphertext Security: definition

$\mathbb{E}$ = (G,E,D) public-key enc. over (M,C). For b=0,1 define EXP(b):



Construction of IND-CCA secure PKE

The Twin Elgamal system

- $\mathbb{G}$: finite cyclic group of prime order p with generator g
- $(\mathsf{Enc}', \mathsf{Dec}')$: AE scheme with keyspace $\mathscr{K}$
- New ingredient: "Random"-ish hash function $H : \mathbb{G}^2 \to \mathscr{K}$

<u>$\mathsf{Gen}(1^n)$:</u>

1. Sample $a_1, a_2 \leftarrow \mathbb{Z}_p^*$
2. Set $A_1 = g^{a_1}, A_2 = g^{a_2}$
3. Output $(\mathsf{sk} = (a_1, a_2), \mathsf{pk} = (A_1, A_2))$

<u>$\mathsf{Enc}(\mathsf{pk}, m)$:</u>

1. Sample $b \leftarrow \mathbb{Z}_p^*$
2. Set $k := H(A_1^b, A_2^b)$
3. Set $c \leftarrow \mathsf{Enc}(k, m)$
4. Output $c' = (g^b, c)$

<u>$\mathsf{Dec}(\mathsf{sk} = (a_1, a_2), (B, c))$:</u>

1. Compute $k = H(B^{a_1}, B^{a_2})$
2. Output $m = \mathsf{Dec}'(k, c)$

Security of Twin Elgamal

Security Theorem:

- If CDH holds in the group $\mathbb{G}$,
- $(\mathsf{Enc}', \mathsf{Dec}')$ is an AE scheme, and
- $H : \mathbb{G}^2 \to \mathscr{K}$ is a "random oracle"

then twin ElGamal is CCA^{ro} secure.

Cost: one more exponentiation during enc/dec

ElGamal security w/o random oracles?

Can we prove CCA security without random oracles?

- Option 1: use Hash-DH assumption in “bilinear groups”
  - Special elliptic curve with more structure [CHK’04 + BB’04]

- Option 2: use Decision-DH assumption in any group [CS’98]

Further Reading

- The Decision Diffie-Hellman problem. D. Boneh, ANTS 3, 1998
- Universal hash proofs and a paradigm for chosen ciphertext secure public key encryption. R. Cramer and V. Shoup, Eurocrypt 2002
- Chosen-ciphertext security from Identity-Based Encryption.
  D. Boneh, R. Canetti, S. Halevi, and J. Katz, SICOMP 2007
- The Twin Diffie-Hellman problem and applications.
  D. Cash, E. Kiltz, V. Shoup, Eurocrypt 2008
- Efficient chosen-ciphertext security via extractable hash proofs.
  H. Wee, Crypto 2010

Further reading

- A Computational Introduction to Number Theory and Algebra,
  V. Shoup, 2008 (V2), Chapter 1-4, 11, 12

  Available at **//shoup.net/ntb/ntb-v2.pdf**